Concepts in Valuation and Valuation of Securities (Shares and Bonds)

THE TIME VALUE OF MONEY

Introduction

To keep pace with the increasing competition, companies have to go in for new ideas implemented through new projects be it for expansion, diversification or modernization. A project is an activity that involves investing a sum of money now in anticipation of benefits spread over a period of time in the future. How do we determine whether the project is financially viable or not? Our immediate response to this question will be to sum up the benefits accruing over the future period and compare the total value of the benefits with the initial investment. If the aggregate value of the benefits exceeds the initial investment, the project is considered to be financially viable.

While this approach *prima facie* appears to be satisfactory, we must be aware of an important assumption that underlies. We have assumed that irrespective of the time when money is invested or received, the value of money remains the same. Put differently, we have assumed that: value of one rupee now = value of one rupee at the end of year 1 = value of one rupee at the end of year 2 and so on. We know intuitively that this assumption is incorrect because money has time value. How do we define this time value of money and build it into the cash flows of a project? The answer to this question forms the subject matter of this chapter.

We intuitively know that ` 1,000 in hand now is more valuable than ` 1,000 receivable after a year. In other words, we will not part with ` 1,000 now in return for a firm assurance that the same sum will be repaid after a year. But we might part with ` 1,000 now if we are assured that something more than ` 1,000 will be paid at the end of the first year. This additional compensation required for parting with ` 1,000 now is called 'interest' or the time value of money. Normally, interest is expressed in terms of percentage per annum for example, 12 per cent p.a. or 18 per cent p.a. and so on.

Why should money have time value? Here are some important reasons for this phenomenon:

Money can be employed productively to generate real returns. For instance, if a sum of ` 100 invested in raw material and labor results in finished goods worth ` 105, we can say that the investment of ` 100 has earned a rate of return of 5 per cent.

In an inflationary period, a rupee today has a higher purchasing power than a rupee in the future.

Since future is characterized by uncertainty, individuals prefer current consumption to future consumption.

The manner in which these three determinants combine to determine the rate of interest can be symbolically represented as follows:

Nominal or market interest rate

= Real rate of interest or return + Expected rate of inflation

+ Risk premiums to compensate for uncertainty

There are two methods by which the time value of money can be taken care of – compounding and discounting. To understand the basic ideas underlying these two methods, let us consider a project which involves an immediate outflow of say ` 1,000 and the following pattern of inflows:

Year 1: ` 250

Year 2: ` 500

Year 3: ` 750

Year 4: ` 750

The initial outflow and the subsequent inflows can be represented on a time line as given below:

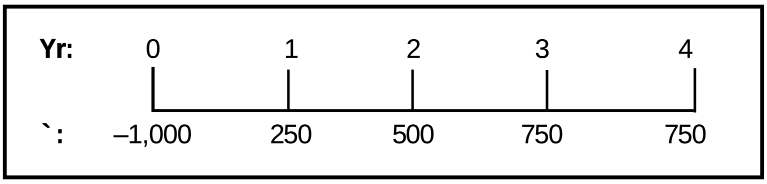

Figure 3.1: Time Line

Process of Compounding

Under the method of compounding, we find the future values (FV) of all the cash flows at the end of the time horizon at a particular rate of interest. Therefore, in this case we will be comparing the future value of the initial outflow of ` 1,000 as at the end of year 4 with the sum of the future values of the yearly cash inflows at the end of year 4. This process can be schematically represented as follows:

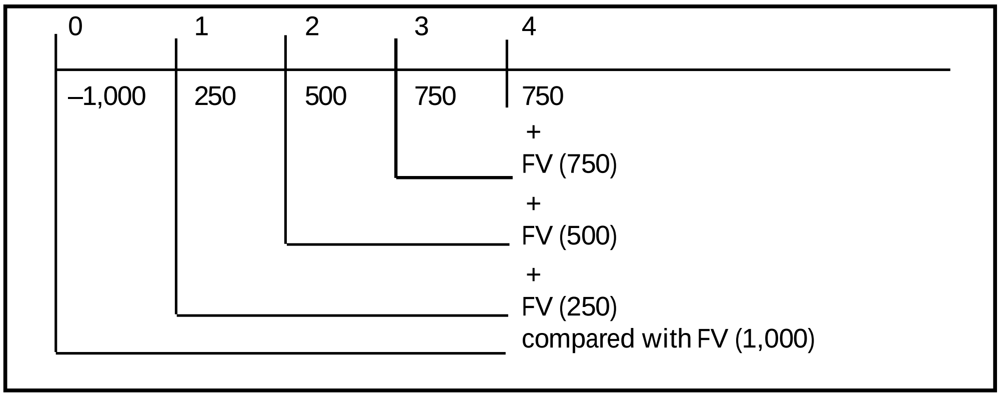

Figure 3.2: Process of Compounding

Under this method of compounding, the future values of all cash inflows at the end of the time horizon at a particular rate of interest are found. Interest is compounded when the amount earned on an initial deposit becomes part of the principal at the end of the first compounding period. If Mr. A invests ` 1,000 in a bank which offers him 5% interest compounded annually, he has ` 1,050 in his account at the end of the first year. The total of the interest and principal ` 1,050 constitutes the principal for the next year. He thus earns ` 1,102.50 for the second year. This becomes the principal for the third year. This compounding procedure will continue for an indefinite number of years. The compounding of interest can be calculated by the following equation:

$$A = P(1 + i)^n$$

Where, A = Amount at the end of the period

P = Principal at the end of the period

i = rate of interest

n = number of years

The amount of money in the account at the end of various years is calculated as under, using the equation:

Amount at the end of year 1 = ` 1,000 (1 + 0.05) = ` 1,050

Amount at the end of year 2 = ` 1,050 (1 + 0.05) = ` 1,102.50

Amount at the end of year 3 = ` 1,102.50 (1 + 0.05) = ` 1,157.63

Year	1	2	3
Beginning amount	` 1,000	` 1,050	` 1,102.50
Interest rate	5%	5%	5%
Amount of interest	50	52.50	55.13
Beginning principal	1,000	` 1,050	` 1,102.50
Ending principal	` 1,050	` 1,102.50	` 1,157.63

The amount at the end of year 2 can be ascertained by substituting

` 1000 (1 + 0.05) for

` 1,050, that is, ` 1,000(1 + 0.05) (1 + 0.05) = ` 1,102.50.

Similarly, the amount at the end of year 3 can be ascertained by substituting

` 1,000(1 + 0.05) (1 + 0.05) (1 + 0.05) = ` 1,157.63.

Thus by substituting the actual figures for the investment or ` 1,000 in the formula $\boldsymbol{A = P(1 + i)^n}$, we arrive at the result shown above in Table.

Process of Discounting

Under the method of discounting, we reckon the time value of money now i.e. at time 0 on the time line. So, we will be comparing the initial outflow with the sum of the present values (PV) of the future inflows at a given rate of interest. This process can be diagrammatically represented as follows:

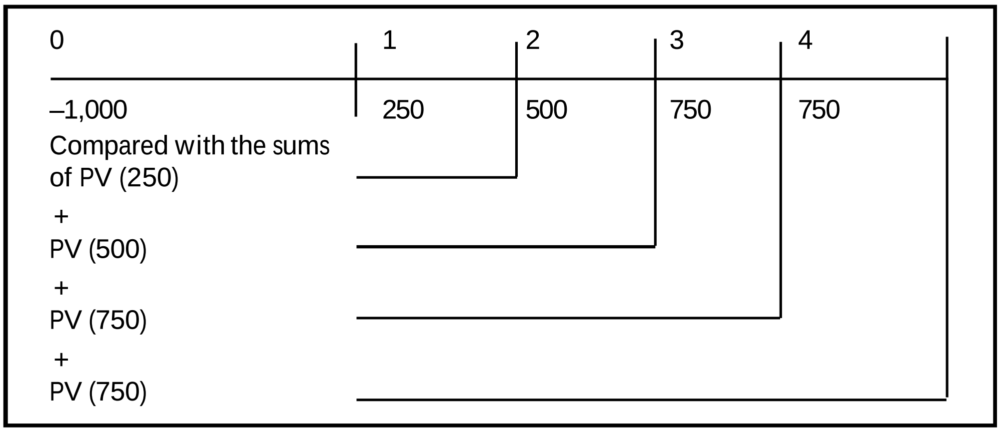

Figure 3.3: Process of Discounting

How do we compute the future values and the present values? This question is answered in the latter part of the chapter. But before that, we must draw the distinction between the concepts of compound interest and simple interest. We shall illustrate this distinction through the following illustration.

Under the method of discounting, we find the time value of money now, that is, at time 0 on the time line. It is concerned with determining the present value of a future amount. This is in contrast to the compounding approach where we convert present amounts into future amounts; in discounting approach we convert the future value to present sums. For example, if Mr. A requires to have ` 1,050 at the end of year 1, given the rate of interest as 5%, he would like to know how much he should invest today to earn this amount. If P is the unknown amount and using the equation we get P (1 + 0.5) = 1,050. Solving the equation, we get P = ` 1,050/1.05 = ` 1,000.

Thus ` 1,000 would be the required principal investment to have ` 1,050 at the end of year 1 at 5% interest rate. In other words, the present value of ` 1,050 received one year from now, rate of interest 5%, is ` 1,000. The present value of money is the reciprocal of the compounding value. Mathematically, we have $P = A \{1/(1 + i)^n\}$ in which P is the present value for the future sum to be received, A is the sum to be received in future, i is the interest rate and n is the number of years.

Illustration 1. If X has a sum of ` 1,000 to be invested, and there are two schemes, one offering a rate of interest of 10 per cent, compounded annually, and other offering a simple rate of interest of 10 per cent, which one should he opt for assuming that he will withdraw the amount at the end of (a) one year (b) two years, and (c) five years?

Solution: Given the initial investment of ` 1,000, the accumulations under the two schemes will be as follows:

End of year	Compounded Interest Scheme	Simple Interest Scheme
1	1000 + (1000 × 0.10) = 1,100	1000 + (1000 × 0.10) = 1,100
2	1100 + (1100 × 0.10) = 1,210	1100 + (1000 × 0.10) = 1,200
3	1210 + (1210 × 0.10) = 1,331	1200 + (1000 × 0.10) = 1,300
4	1331 + (1331 × 0.10) = 1,464	1300 + (1000 × 0.10) = 1,400
5	1464 + (1464 × 0.10) = 1,610	1400 + (1000 × 0.10) = 1,500

From this table, it is clear that under the compound interest scheme interest earns interest, whereas interest does not earn any additional interest under the simple interest scheme. Obviously, an investor seeking to maximize returns will opt for the compound interest scheme if his holding period is more than a year. We have drawn the distinction between compound interest and simple interest here to emphasize that in financial analysis we always assume interest to be compounded.

Future Value of a Single Flow (Lump Sum)

The above table illustrates the process of determining the future value of a lump sum amount invested at one point of time. But the way it has gone about calculating the future value will prove to be cumbersome if the future value over long maturity periods of 20 years or 30 years is to be calculated. A generalized procedure for calculating the future value of a single cash flow compounded annually is as follows:

$$FV_n = PV (1 + k)^n$$

where,

FV_n = Future value of the initial flow n years hence

PV = Initial cash flow

k or i = Annual rate of interest

n = Life of investment

In the above formula, the expression $(1 + k)^n$ represents the future value of an initial investment of Re.1 (one rupee invested today) at the end of n years at a rate of interest k referred to as Future Value Interest Factor (FVIF, hereafter). To simplify calculations, this expression has been evaluated for various combinations of k and n and these values are presented in Table 1 at the end of this book. To calculate the future value of any investment for a given value of 'k' and 'n', the corresponding value of $(1 + k)^n$ from the table has to be multiplied with the initial investment.

Illustration 2. The fixed deposit scheme of Andhra Bank offers the following interest rates.

Period of Deposit	Rate per Annum
46 days to 179 days	10.0%
180 days to < 1 year	10.5%
1 year and above	11.0%

An amount of ` 10,000 invested today will grow in 3 years to

Solution: $FV_n = PV(1 + k)^n$

$= PV \times FVIF\ (11, 3)$

$= 10{,}000\ (1.368)$

= ` 13,680

Illustration 3. The fixed deposit scheme of a bank offers the following interest rates:

Period of Deposit	Rate per Annum
< 45 days	9%
46 days to 179 days	10%
180 days to 365 days	10.5%
366 days and above	12%

How much does an investment of ` 10,000 invested today grow to in 1 years?

Solution: $FV_n = PV(1 + i)^n$ or $PV \times FVIF(12\%, 3y)$

$= 10{,}000 \times 1.4049$ (from the tables)

= ` 14,049

Doubling Period: A frequent question posed by the investor is, "How long will it take for the amount invested to be doubled for a given rate of interest". This question can be answered by a rule known as 'rule of 72'. Though it is a crude way of calculating this rule says that the period within which the amount will be doubled is obtained by dividing 72 by the rate of interest.

For instance, if the given rate of interest is 6 per cent, then doubling period is 72/6 = 12 yrs.

However, an accurate way of calculating doubling period is the 'rule of 69', according to which, doubling period

$$= 0.35 + \frac{69}{\text{Interest rate}}$$

Illustration 4. The following is the calculation of doubling period for two rates of interest i.e., 6 per cent and 12 per cent.

Solution:

Rate of interest	Doubling Period
6%	= 0.35 + 69/6 = 0.35 + 11.5 = 11.85 yrs.
12%	= 0.35 + 69/12 = 0.35 + 5.75 = 6.1 yrs.

Growth Rate: The compound rate of growth for a given series for a period of time can be calculated by employing the future value interest factor table (FVIF).

Illustration 5.

Years	1	2	3	4	5	6
Profits (in lakh)	95	105	140	160	165	170

How is the compound rate of growth for the above series determined? This can be done in two steps:

Solution:

The ratio of profits for year 6 to year 1 is to be determined i.e., 170/95 = 1.79

The $FVIF_{k,n}$ table is to be looked at. Look at a value which is close to 1.79 for the row for 5 years. The value close to 1.79 is 1.762 and the interest rate corresponding to this is 12 per cent. Therefore, the compound rate of growth is 12 per cent.

Increased Frequency of Compounding: In the above illustration, the compounding has been done annually. Suppose we are offered a scheme where compounding is done more frequently. For example, assume you have deposited ` 10,000 in a bank which offers 10 per cent interest per annum compounded semi-annually which means that interest is paid every six months.

	`
Now, amount in the beginning	= 10,000
Interest @ 10 per cent p.a. for first six months	= 500
Amount at the end of six months $\left(10{,}000 \times \frac{0.1}{2}\right)$	= 10,500
Interest for second 6 months $\left(10{,}500 \times \frac{0.1}{2}\right)$	= 525
Amount at the end of the year	= 11,025

Instead, if the compounding is done annually, the amount at the end of the year will be 10,000 (1 + 0.1) = ` 11,000. This difference of ` 25 is because under semi-annual compounding, the interest for first 6 months earns interest in the second 6 months.

The generalized formula for these shorter compounding periods is:

$$FV_n = PV\left(1+\frac{k}{m}\right)^{m\times n}$$

Where,

FV_n = Future value after 'n' years

PV = Cash flow today

k or i = Nominal interest rate per annum

m = Number of times compounding is done during a year

n = Number of years for which compounding is done.

Illustration 6. Under the Vijaya Cash Certificate scheme of Vijaya Bank, deposits can be made for periods ranging from 6 months to 10 years. Every quarter, interest will be added on to the principal. The rate of interest applied is 9 per cent p.a. for periods from 12 to 23 months and 10 per cent p.a. for periods from 24 to 120 months.

Solution: An amount of ` 1,000 invested for 2 years will grow to

$$FV_n = PV\left(1+\frac{k}{m}\right)^{m\times n}$$

where m = frequency of compounding during a year

$$= 1{,}000\left(1+\frac{0.10}{4}\right)^{8}$$

$= 1{,}000(1.025)^8$

$= 1{,}000 \times 1.2184 =$ ` 1,218

Illustration 7. Under the Andhra Bank's Cash Multiplier Scheme, deposits can be made for periods ranging from 3 months to 5 years. Every quarter, interest is added to the principal. The applicable rate of interest is 9% for deposits less than 23 months and 10% for periods more than 24 months. What will the amount of ` 10,000 today be after 2 years?

Solution: $FV_n = PV(1 + i/m)^{m \times n}$

$= 1{,}000\ (1 + 0.10/4)^{4 \times 2}$

$= 1{,}000\ (1 + 0.10/4)^{8}$

= ` 12,180

Effective vs. Nominal Rate of Interest: We have seen above that the accumulation under the semi-annual compounding scheme exceeds the accumulation under the annual compounding scheme by ` 25. This means that while under annual compounding scheme, the nominal rate of interest is 10 per cent per annum, under the scheme where compounding is done semi-annually, the principal amount grows at the rate of 10.25 per cent per annum. This 10.25 per cent is called the effective rate of interest which is the rate of interest per annum under annual compounding that produces the same effect as that produced by an interest rate of 10 per cent under semi-annual compounding.

The general relationship between the effective and nominal rates of interest is as follows:

$$r = \left(1+\frac{k}{m}\right)^{m} - 1$$

where, r = Effective rate of interest

k = Nominal rate of interest

m = Frequency of compounding per year

Illustration 8. Find out the effective rate of interest, if the nominal rate of interest is 12 per cent and is quarterly compounded.

Solution: Effective rate of interest

$$r = \left(1+\frac{k}{m}\right)^{m} - 1$$

$$r = \left(1+\frac{0.12}{4}\right)^{4} - 1$$

$$= (1 + 0.03)^{4} - 1 = 1.126 - 1$$

$$= 0.126 = 12.6\% \text{ p.a.}$$

Future Value of Multiple Flows: Suppose we invest ` 1,000 now (beginning of year 1), ` 2,000 at the beginning of year 2 and ` 3,000 at the beginning of year 3, how much will these flows accumulate to at the end of year 3 at a rate of interest of 12 per cent per annum? This problem can be represented on the time line as follows:

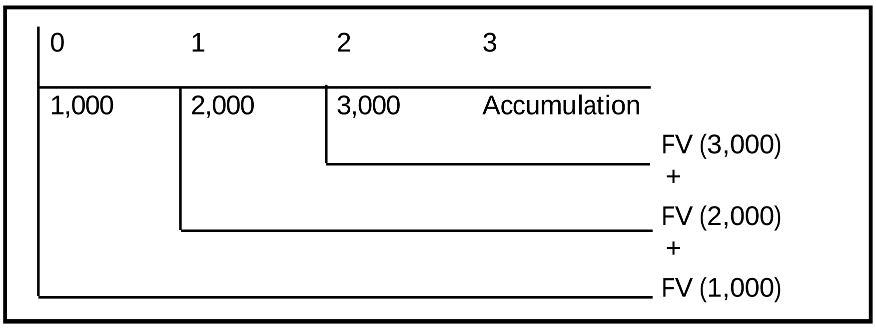

Figure 4: Compounding Process for Multiple Flows

To determine the accumulated sum at the end of year 3, we have to just add the future compounded values of ` 1,000, ` 2,000 and ` 3,000 respectively *.

FV ` 1,000 FV ` 2,000 FV ` 3,000

At k = 0.12, the above sum is equal to

= ` 1,000 × $FVIF_{12,3}$ 2,000 × $FVIF_{12,2}$ 3,000 × $FVIF_{12,1}$

= ` [1,000 × 1.405 2,000 × 1.254 3,000 × 1.120] = ` 7,273

Therefore, to determine the accumulation of multiple flows as at the end of a specified time horizon, we have to find out the accumulations of each of these flows using the appropriate FVIF and sum up these accumulations. This process can get tedious if we have to determine the accumulation of multiple flows over a long period of time, for example, the accumulation of a recurring deposit of ` 100 per month for 60 months at a rate of 1 per cent per month. In such cases a short cut method can be employed provided the flows are of equal amounts. This method is discussed in the following section.

Illustration 9. We have considered only single payment made once and its accumulation effect. An investor may be interested in investing money in installments and wish to know the value of his savings after n years. For example, Mr. Madan invests ` 500, ` 1,000, ` 1,500, ` 2,000 and ` 2,500 at the end of each year for 5 years. Calculate the value at the end of 5 years compounded annually if the rate of interest is 5% p.a.

Solution:

End of Year	Amount Investment	Number of Years Compounded	Compounded Interest Factor from Tables	FV in `
1	` 500	4	1,216	608
2	` 1,000	3	1,158	1,158
3	` 1,500	2	1,103	1,654
4	` 2,000	1	1,050	2,100
5	` 2,500	0	1,000	2,500
Amount at the end of 5th Year				` 8,020

Future Value of Annuity: Annuity is the term used to describe a series of periodic flows of equal amounts. These flows can be either receipts or payments. For example, if you are required to pay ` 200 per annum as life insurance premium for the next 20 years, you can classify this stream of payments as an annuity. If the equal amounts of cash flow occur at the end of each period over the specified time horizon, then this stream of cash flows is defined as a regular annuity or deferred annuity. When cash flows occur at the beginning of each period the annuity is known as an annuity due.

The future value of a regular annuity for a period of n years at a rate of interest ‘k’ is given by the formula:

$FVA_n = A(1 + k)^{n-1} + A(1 + k)^{n-2} + A(1 + k)^{n-3} + \ldots\ldots + A$

which reduces to

$$FVA_n = A\left[\frac{(1+k)^n - 1}{k}\right]$$

where, A = Amount deposited/invested at the end of every year for n years

k or i = Rate of interest (expressed in decimals)

n = Time horizon

FVA_n = Accumulation at the end of n years

The expression $\left[\frac{(1+k)^n - 1}{k}\right]$ is called the Future Value Interest Factor for Annuity (FVIFA, *Candidates who would like to know whether there is any short cut for evaluating $(1 + k)^n$ for values of 'k' not found in the table, are informed that there is no short cut method except using logarithms or the X^Y function found in scientific calculators.

Illustration 10. M. Ram Kumar deposits ` 3,000 at the end of every year for 5 years into his account for 5 years, interest being 5% compounded annually. Determine the amount of money he will have at the end of the 5th year.

End of Year	Amount Investment	Number of Years Compounded	Compounded Interest Factor from Tables	FV in `
1	` 2,000	4	1,216	2,432
2	` 2,000	3	1,158	2,316
3	` 2,500	2	1,103	2,206
4	` 2,000	1	1,050	2,100
5	` 2,500	0	1,000	2,000
Amount at the end of 5th Year				` 11,054

OR Using formula and the tables we can find that: = 2000 FVIFA(5%, 5y)

= 2,000 × 5.526

= **` 11,052**

We notice that we can get the accumulations at the end of n period using the tables. Calculations for a long time horizon are easily done with the help of reference tables. Annuity tables are widely used in the field of investment banking as ready reckoners.

Illustration 11. Calculate the value of an annuity flow of ` 5,000 done on a yearly basis for 5 years, yielding an interest of 8% p.a.

Solution: = 5000 FVIFA(8%, 5y)

= 5,000 × 5.867

= ` 29,335

Illustration 12. Under the recurring deposit scheme of the Vijaya Bank, a fixed sum is deposited every month on or before the due date opted for 12 to 120 months according to the convenience and needs of the investor. The period of deposit, however, should be in multiples of 3 months only. The rate of interest applied is 9 per cent p.a. for periods from 12 to 24 months and 10 per cent p.a. for periods from 24 to 120 months and is compounded at quarterly intervals.

Solution: Based on the above information the maturity value of a monthly installment of ` 5 for 12 months can be calculated as below:

Amount of deposit = ` 5 per month

Rate of interest = 9 per cent p.a. compounded quarterly

Effective rate of interest per annum = $\left(1+\frac{0.09}{4}\right)^4 - 1 = 0.0931$

Rate of interest per month $= (r + 1)^{1/m} - 1$

$= (1 + 0.0931)^{1/12} - 1$

$= 1.0074 - 1 = 0.0074 = 0.74\%$

Maturity value can be calculated using the formula

$$FVA_n = A\left\{\frac{(1+k)^n - 1}{k}\right\}$$

$$= 5\left\{\frac{(1+0.0074)^{12} - 1}{0.0074}\right\}$$

= 5 × 12.50 = ` 62.50

If the payments are made at the beginning of every year, then the value of such an annuity called annuity due is found by modifying the formula for annuity regular as follows:

FVA_n (due) = A (1 + k) $FVIFA_{k,n}$

Illustration 13. Under the Jeevan Mitra Plan offered by Life Insurance Corporation of India, if a person is insured for ` 10,000 and if he survives the full term, then the maturity benefits will be the basic sum of ` 10,000 assured plus bonus which accrues on the basic sum assured. The minimum and maximum age to propose for a policy is 18 and 50 years respectively.

Let us take two examples, one of a person aged 20 and another of 40 years old to illustrate this scheme.

The person aged 20, enters the plan for a policy of ` 10,000. The term of policy is 25 years and the annual premium is ` 41.65. The person aged 40, also proposes for the policy of ` 10,000 and for 25 years and the annual premium he has to pay comes to ` 57. What are the rates of return enjoyed by these two persons?

Solution: Rate of return enjoyed by the person of 20 years of age

Premium = ` 41.65 per annum

Term of Policy = 25 years

Maturity Value = ` 10,000 + bonus which can be overlooked as it is a fixed amount and does not vary with the term of policy.

We know that the premium amount when multiplied by FVIFA factor will give us the value at maturity.

i.e., $P \times (1 + k)\ \text{FVIFA}\ (k, n) = MV$

where,

P = Annual premium

n = Term of policy in years

k = Rate of return

MV = Maturity value

Therefore,

$41.65 \times (1 + k)\ \text{FVIFA}\ (k,25) = 10{,}000$

$(1 + k)\ \text{FVIFA}\ (k,25) = 240.01$

From table 2 at the end of the book, we can find that

$(1 + 0.14)\ \text{FVIFA}\ (14,25) = 207.33$

i.e., $(1.14)\ \text{FVIFA}\ (14,25) = 1.14 \times 181.871 = 207.33$

and

$(1 + 0.15)\ \text{FVIFA}\ (15,25) = 244.71$

i.e., $(1.15)\ \text{FVIFA}\ (15,25) = 1.15 \times 212.793 = 244.71$

By interpolation:

$$k = 14\% + (15\% - 14\%) \times \frac{240.01 - 207.33}{244.71 - 207.33}$$

$$= 14\% + 1\% \times \frac{33.68}{33.38}$$

$$= 14\% + 0.87\% = 14.87\%$$

Rate of return enjoyed by the person aged 40

Premium = ` 57 per annum

Term of Policy = 25 years

Maturity Value = ` 10,000

Therefore, 57 × (1 + k) FVIFA (k,25) = 10,000

(1 + k) FVIFA (k,25) = 175.44

From table 2 at the end of the book, we can find that

(1 + k) FVIFA (13%, 25) = 175.87

i.e., (1.13) (155.62) = 175.87

i.e., k = 13% (approx.)

Here we find that the rate of return enjoyed by the 20-year old person is greater than that of the 40-year old person by about 2 per cent in spite of the latter paying a higher amount of annual premium for the same period of 25 years and for the same maturity value of ` 10,000. This is due to the coverage for the greater risk in the case of the 40-year old person.

Now that we are familiar with the computation of future value, we will get into the mechanics of computation of present value.

Sinking Fund Factor

We have the equation

$$FVA = A\left[\frac{(1+k)^n - 1}{k}\right]$$

We can rewrite it as

$$A = FVA\left[\frac{k}{(1+k)^n - 1}\right]$$

The expression $\left[\frac{k}{(1+k)^n - 1}\right]$ is called the Sinking Fund Factor. It represents the amount that has to be invested at the end of every year for a period of "n" years at the rate of interest "k", in order to accumulate Re.1 at the end of the period.

Discounting or Present Value of a Single Flow

Discounting as explained earlier is an alternative approach for reckoning the time value of money. Using this approach, we can determine the present value of a future cash flow or a stream of future cash flows. The present value approach is the commonly followed approach for evaluating the financial viability of projects.

If we invest ` 1,000 today at 10 per cent rate of interest for a period of 5 years, we know that we will get ` 1,000 × FVIF (10,5) = ` 1,000 × 1.611 = ` 1,611 at the end of 5 years. The sum of ` 1,611 is called the accumulation of ` 1,000 for the given values of 'k' and 'n'. Conversely, the sum of ` 1,000 invested today to get ` 1,611 at the end of 5 years is called the present value of ` 1,611 for the given values of 'k' and 'n'. It, therefore, follows that to determine the present value of a future sum we have to divide the future sum by the FVIF value corresponding to the given values of 'k' and 'n' i.e. present value of ` 1,611 receivable at the end of 5 years at 10 per cent rate of interest.

$$= \text{\`}\ \frac{1{,}611}{\text{FVIF}(10{,}5)} = \text{\`}\ \frac{1{,}611}{1.611} = \text{\`}\ 1{,}000$$

In general the present value (PV) of a sum (FV_n) receivable after n years at a rate of interest (k) is given by the expression.

$$PV = \frac{FV_n}{\text{FVIF}(k, n)} = \frac{FV_n}{(1+k)^n}$$

The inverse of FVIF (k,n) is defined as PVIF (k, n) (Present Value Interest Factor for k,n). Therefore, the above equation can be written as

$$PV = FV_n \times \text{PVIF}(k,n)$$

Therefore to determine the present value of a future sum, we have to just locate the PVIF factor for the given values of k and n and multiply this factor value with the given sum. Since PVIF (k,n) represents the present value of Re. 1 receivable after n years at a rate of interest k, it is obvious that PVIF values cannot be greater than one. The PVIF values for different combinations of k and n are given in table 3 at the end of this book.

Illustration 14. Calculate the PV of an annuity of ` 500 received annually for 4 year, when discounting factor is 10%.

End of Year	Cash Inflows	PV Factor	PV in `
1	` 500	0.909	454
2	` 500	0.827	413
3	` 500	0.751	375
4	` 500	0.683	341

Present Value of an annuity ` 1,585.

OR by directly looking at the table we can calculate: = 500 × PVIFA(10%, 4y)

= 500 × 3.170

= ` 1,585

Illustration 15. Find out the present value of an annuity of ` 10,000 over 3 years when discounted at 5%.

Solution: = 10,000 × PVIFA(5%, 3y)

= 10,000 × 2.773

= ` 27,730

Illustration 16. The cash certificates of Andhra Bank are a term deposit scheme under reinvestment plan. Interest on deposit money earns interest as it is reinvested at quarterly rests. These deposits suit depositors from lower and middle income groups, since the small odd sums invested grow into large amounts over a period of time.

Solution: Given an interest rate of 12 per cent p.a. on a certificate having a value of ` 100 after 1 year, the issue price of the cash certificate can be calculated as below.

The effective rate of interest has to be calculated first.

$$r = \left(1+\frac{k}{m}\right)^m - 1$$

$$r = \left(1+\frac{0.12}{4}\right)^4 - 1 = 12.55\%$$

The issue price of the cash certificate is

$$PV = \frac{FV_n}{(1+k)^n}$$

$$= \frac{100}{(1+0.1255)^1} = \text{\`{}}\ 88.85$$

Illustration 17. Pragati cash certificate scheme of Syndicate Bank is an ideal scheme for all classes of people under different income groups. A small odd sum can be invested for a period ranging from 1 to 10 years. The certificates are issued in convenient denominations of ` 25, ` 100, ` 1,000, and ` 1,00,000. The rate of interest is 12 per cent p.a. compounded quarterly.

Solution: To calculate the issue price of a certificate of ` 1,00,000 to be received after 10 years, the following formula can be used

$$PV = \frac{FV_n}{(1+k)^n}$$

Firstly, the effective rate of interest has to be calculated.

$$r = \left(1+\frac{0.12}{4}\right)^4 = 12.55\%$$

The issue price of the cash certificate can now be calculated as:

$$PV = \frac{FV_n}{(1+k)^n}$$

$$= \frac{1,00,000}{(1+0.1255)^{10}} = \text{\`{}}\ 30,658$$

Present Value of Uneven Multiple Flows

Suppose a project involves an initial investment of ` 10 lakh and generates net inflows as follows:

End of Year -> 1 ` 2 lakh

-> 2 ` 4 lakh

-> 3 ` 6 lakh

What is the present value of the future cash inflows? To determine it, we have to first define the relevant rate of interest. The relevant rate of interest as we shall see later, will be the cost of the funds invested. Suppose, we assume that this cost is 12 per cent p.a. then we can determine the present value of the cash flows using the following two-step procedure:

Step 1

Evaluate the present value of cash inflow independently. In this case, the present values will be as follows:

Year	Cash Flow (` in lakh)	Present Value (` in lakh)
1	2	2 × PVIF (12,1) = 2 × 0.893 = 1.79
2	4	4 × PVIF (12,2) = 4 × 0.797 = 3.19
3	6	6 × PVIF (12,3) = 6 × 0.712 = 4.27

Step 2

Aggregate the present values obtained in Step 1 to determine the present value of the cash flow stream. In this case the present value of the cash inflows associated with the project will be ` (1.79 + 3.19 + 4.27) lakh = ` 9.25 lakh.

A project is said to be financially viable if the present value of the cash inflows exceeds the present value of the cash outflow. In this case, the project is not financially viable because the present value of the net cash inflows (` 9.25 lakh) is less than the initial investment of ` 10 lakh. The difference of ` 0.75 lakh is called the net present value.

Like the procedure followed to obtain the future value of multiple cash flows, the procedure adopted to determine the present value of a series of future cash flows can prove to be cumbersome, if the time horizon to be considered is quite long. These calculations can, however, be simplified if the cash flows occurring at the end of the time periods are equal. In other words, if the stream of cash flows can be regarded as a regular annuity or annuity due, then the present value of this annuity can be determined using an expression similar to the FVIFA expression.

Illustration 18. An investor will receive ` 10,000, ` 15,000, ` 8,000,

` 11,000 and ` 4,000 respectively at the end of each of 5 years. Find out the present value of this stream of uneven cash flows, if the investor's interest rate is 8%.

PV = $10{,}000/(1 + 0.08) + 15{,}000/(1 + 0.08)^2 + 8{,}000/(1 + 0.08)^3 + 11{,}000/(1 + 0.08)^4 + 4{,}000/(1 + 0.08)^5$

= ` 39,276 **Or**

PV = 10,000 PVIF(8,1) + 15,000 PVIF(8,2) + 8,000 PVIF(8,3) + 11,000 PVIF(8,4) + 4,000 PVIF(8,5)

= $10{,}000 \times 0.926 + 15{,}000 \times 0.857 + 8{,}000 \times 0.794 + 11{,}000 \times 0.735 + 4{,}000 \times 0.681$

= ` 39,276

Present Value of an Annuity

The present value of an annuity 'A' receivable at the end of every year for a period of n years at a rate of interest k is equal to

$$PVA_n = \frac{A}{(1+k)} + \frac{A}{(1+k)^2} + \frac{A}{(1+k)^3} + \ldots\ldots + \frac{A}{(1+k)^n};$$

which reduces to

$$PVA_n = A \times \left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$$

The expression $\left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$ is called the PVIFA (Present Value Interest Factor Annuity) and it represents the present value of a regular annuity of Re. 1 for the given values of k and n. The values of PVIFA (k, n) for different combinations of 'k' and 'n' are given in Table 4 given at the end of the book. It must be noted that these values can be used in any present value problem only if the following conditions are satisfied: (a) the cash flows are equal; and (b) the cash flows occur at the end of every year. It must also be noted that PVIFA (k, n) is not the inverse of FVIFA (k, n) although PVIF (k, n) is the inverse of FVIF (k, n). The following illustration illustrates the use of PVIFA tables for determining the present value.

Illustration 19. The Swarna Kalash Yojana at rural and semi-urban branches of SBI is a scheme open to all individuals/firms. A lump sum deposit is remitted and the principal is received with interest at the rate of 12 per cent p.a. in 12 or 24 monthly installments. The interest is compounded at quarterly intervals.

Solution: The amount of initial deposit to receive a monthly installment of ` 100 for 12 months can be calculated as below:

Firstly, the effective rate of interest per annum has to be calculated.

$$r = \left(1 + \frac{k}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.12}{4}\right)^4 - 1 = 12.55\%$$

After calculating the effective rate of interest per annum, the effective rate of interest per month has to be calculated which is nothing but

$(1.1255)^{1/12} - 1 = 0.00990$

The initial deposit can now be calculated as below:

$$PVA_n = A \times \left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$$

$$= 100 \times \left[\frac{(1+0.00990)^{12} - 1}{0.00990(1+0.00990)^{12}}\right]$$

$$= 100 \times \left[\frac{0.1255}{0.01114}\right]$$

= 100 × 11.26 = ` 1,126.

Illustration 20. The annuity deposit scheme of SBI provides for fixed monthly income for suitable periods of the depositor's choice. An initial deposit has to be made for a minimum period of 36 months. After the first month of the deposit, the depositor receives monthly installments depending on the number of months he has chosen as annuity period. The rate of interest is 11 per cent p.a. which is compounded at quarterly intervals.

Solution: If an initial deposit of ` 4,610 is made for an annuity period of 60 months, the value of the monthly annuity can be calculated as below.

Firstly, the effective rate of interest per annum has to be calculated

$$r = \left(1+\frac{k}{m}\right)^m - 1$$

$$= \left(1+\frac{0.11}{4}\right)^4 - 1 = 11.46\%$$

After calculating the effective rate of interest per annum, the effective rate of interest per month has to be calculated which is nothing but

$(1.1146)^{1/12} - 1 = 0.00908$

The monthly annuity can now be calculated as

$$PVA_n = A \times \left[\frac{(1+k)^n - 1}{k(1+k)^n}\right]$$

$$4{,}610 = A \times \left[\frac{(1+0.00908)^{60} - 1}{0.00908(1.00908)^{60}}\right]$$

$$4{,}610 = A \times = 99.8833$$

$$\Rightarrow A = 99.8833$$

A = ` 100

Capital Recovery Factor: Manipulating the relationship between PVA_n, A, k & n we get an equation:

$$A = PVA_n \left[\frac{(1+k)^n - 1}{k(1+k)^n} \right]$$

$\left[\frac{(1+k)^n - 1}{k(1+k)^n} \right]$ is known as the capital recovery factor.

Illustration 21. A loan of ` 1,00,000 is to be repaid in five equal annual installments. If the loan carries a rate of interest of 14 per cent p.a. the amount of each installment can be calculated as below.

Solution: If R is defined as the equated annual installment, we are given that

R × PVIFA (14%, 5) = ` 1,00,000

$$\text{Therefore, } R = \frac{\text{` } 1{,}00{,}000}{\text{PVIFA}(14\%, 5)}$$

$$= \frac{\text{` } 1{,}00{,}000}{3.433} = \text{` } 29{,}129$$

Notes:

1. We have introduced in this example the application of the inverse of the PVIFA factor which is called the capital recovery factor. The application of the capital recovery factor helps in answering questions like:
   - What should be the amount paid annually to liquidate a loan over a specified period at a given rate of interest?
   - How much can be withdrawn periodically for a certain length of time, if a given amount is invested today?
2. In this example, the amount of ` 29,129 represents the sum of the principal and interest components. To get an idea of the break-up of each installment between the principal and interest components, the loan repayment schedule is given below:

Year (A)	Equated annual installment (B) (`)	Interest content of (B) (C) (`)	Capital content of (B) [(D) = (B – C)] (`)	Loan outstanding after payment (E) (`)
0	–	–	–	1,00,000
1	29,129	14,000	15,129	84,871
2	29,129	11,882	17,247	67,624
3	29,129	9,467	19,662	47,962
4	29,129	6,715	22,414	25,548
5	29,129	3,577	25,552	–

The interest content of each installment is obtained by multiplying interest rate with the loan outstanding at the end of the immediately preceding year.

As can be observed from this schedule, the interest component declines over a period of time whereas the capital component increases. The loan outstanding at the end of the penultimate year must be equal to the capital content of the last installment but in practice there will be a marginal difference on account of rounding-off errors.

3. The equated annual installment method is usually adopted for fixing the loan ment schedule in a hire purchase transaction. But the financial institutions in India repaylike IDBI, IFCI and ICICI do not follow this scheme of equal periodic amortization. Instead, they stipulate that the loan must be repaid in equal installments. According to this scheme, the principal component of each payment remains constant and the total debt-servicing burden (consisting of principal repayment and interest payment) declines over time.

Sinking Fund: Sinking fund is a fund which is created out of fixed payments each period to accumulate to a future sum after a specified period. The sinking fund factor is useful in determining the annual amount to be put in a fund to repay bonds or debentures or to purchase a fixed asset or a property at the end of a specified period.

$A = FVA \times i/\{(1 + i)^n - 1\}$

$i/\{(1+i)^n-1\}$ is called the *Sinking Fund Factor*.

Present Value of Perpetuity

An annuity of an infinite duration is known as perpetuity. The present value of such perpetuity can be expressed as follows:

$$P = A \times PVIFA_k$$

Where, P = Present value of a perpetuity

A = Constant annual payment

$PVIFA_{k,}$ = Present value interest factor for a perpetuity

Therefore, The value of $PVIFA_k$ is

$$\sum_{t=1}^{\infty} \frac{1}{(1+k)^t} = \frac{1}{k}$$

We can say that PV interest factor of a perpetuity is simply one divided by interest rate expressed in decimal form. Hence, PV of a perpetuity is simply equal to the constant annual payment divided by the interest rate.

Illustration 22. If the principal of a college wants to institute a scholarship of ` 5,000 to a meritorious student in finance every year, find out the PV of investment which would yield ` 5,000 in perpetuity, discounted at 10%.

Solution: P = A/i

= 5,000/0.10

= **` 50,000**

This means he should invest ` 50,000 to get an annual return of ` 5,000.

Illustration 23. What is the future value of a regular annuity of Re. 1.00 earning a rate of 12% interest p.a. for 5 years?

Solution: 1 × FVIFA(12%, 5y) = 1 × 6.353 = ` 6.353

Illustration 23. If a borrower promises to pay ` 20,000 eight years from now in return for a loan of ` 12,550 today, what is the annual interest being offered?

Solution: 20000 × PVIF(k%, 8y) = ` 12,550 K is approximately 6%.

Illustration 24. A loan of ` 5,00,000 is to be repaid in 10 equal installments. If the loan carries 12% interest p.a. what is the value of one installment?

Solution: A × PVIFA(12%, 10y) = 5,00,000 So A = 5,00,000/5.650 = ` 88,492.

Illustration 25. A person deposits ` 25,000 in a bank that pays 6% interest half-yearly. Calculate the amount at the end of 3 years.

Solution: 25,000 × (1+0.06)3 × 2 = 25,000 × 1.194 = ` 29,850

Illustration 26. Find the present value of ` 1,00,000 receivable after 10 years if 10% is the time preference for money.

Solution: 1,00,000 × (0.386) = ` 38,600

VALUATION OF BONDS AND SHARES

Introduction

Valuation is the process of linking risk with returns to determine the worth of an asset. Assets can be real or financial; securities are called financial assets, physical assets are real assets. The ultimate goal of any individual investor is maximization of profits. Investment management is a continuous process requiring constant monitoring. The value of an asset depends on the cash flow it is expected to provide over the holding period. The fact that as on date there is no method by which prices of shares and bonds can be accurately predicted should be kept in mind by an investor before he decides to take an investment decision. The present chapter will help us to know why some securities are priced higher than others. We can design our investment structure by exploiting the variables to maximize our returns.

Ordinary shares are riskier than bonds or debentures and some shares are more risky than others. The investor would therefore commit funds on a share only if he is convinced about the rate of return being commensurate with risk.

Concept of Intrinsic Value: A security can be evaluated by the series of dividends or interest payments receivable over a period of time. In other words, a security can be defined as the present value of the future cash streams – the intrinsic value of an asset is equal to the present value of the benefits associated with it. The expected returns (cash inflows) are discounted using the required return commensurate with the risk. Mathematically, it can be represented by:

$$\mathbf{V_0 = C_1/(1 + i)^1 + C_2/(1 + i)^2 + C_3/(1 + i)^3 + C_n/(1 + i)^n}$$

$$= \mathbf{C_n/(1 + i)^n}$$

Where V_0 = Value of the asset at time zero (t = 0)

P_0 = Present value of the asset

C_n = Expected cash flow at the end of period n

I = Discount rate or required rate of return on the cash flows

N = Expected life of an asset.

Illustration 27. Assuming a discount rate of 10% and the cash flows associated with 2 projects A and B over a 3 year period, determine the value of the assets.

Year	Cash flows of A(`)	Cash flows of B(`)
1	20,000	10,000
2	20,000	20,000
3	20,000	30,000

Solution: Value of asset A = 20,000 PVIFA(10%, 3y)

= 20,000 × 2.487

= **` 49,470**

Value of asset B = 10,000 PVIF (10%, 1) + 20,000 PVIF(10%, 2) + 30,000 PVIF (10%, 3)

= 10,000 × 0.909 + 20,000 × 0.826 + 30,000 × 0.751

= 9,090 + 16,520 + 22,530

= **` 48,140**

Illustration 28. Calculate the value of an asset if the annual cash inflow is ` 5,000 per year for the next 6 years and the discount rate is 16%.

Solution:

Value of the asset = $\mathbf{C_n/(1 + i)^n}$

= $5,000/(1 + 0.16)^6$

Or = 5,000 PVIFA(16%, 6y)

= 5,000 × 3.685

= **` 18,425**

Concepts of Value

Book Value: Book value is an accounting concept. Value is what an asset is worth today in terms of their potential benefits. Assets are recorded at historical cost and these are depreciated over years. Book value may include intangible assets at acquisition cost minus amortized value. The book value of a debt is stated at the outstanding amount. The difference between the book value of assets and liabilities is equal to the shareholders' net worth. (Net worth is the sum total of paid-up capital and reserves and surplus). Book value of a share is calculated by dividing the net worth by the number of shares outstanding.

Replacement Value is the amount a company is required to spend if it were to replace its existing assets in the present condition. It is difficult to find cost of assets presently used by the company.

Liquidation Value is the amount a company can realize if it sold the assets after the winding up of its business. It will not include the value of intangibles as the operations of the company will cease to exist. Liquidation value is generally the minimum value the company might accept if it sold its business.

Going Concern Value is the amount a company can realize if it sells its business as an operating one. This value is higher than the liquidation value.

Market Value is the current price at which the asset or security is being sold or bought in the market. Market value per share is generally higher than the book value per share for profitable and growing firms.

Valuation of Bonds: Bonds are long term debt instruments issued by government agencies or big corporate houses to raise large sums of money. Bonds issued by government agencies are secured and those issued by private sector companies may be secured or unsecured. The rate of interest on bonds is fixed and they are redeemable after a specific period. Some important terms in bond valuation:

Face Value: Also known as *par value*, this is the value stated on the face of the bond. It represents the amount that the unit borrows which is to be repaid at the time of maturity, after a certain period of time. A bond is generally issued at values such as ` 100 or ` 1000.

Coupon Rate is the specified rate of interest in the bond. The interest payable at regular intervals is the product of the par value and the coupon rate broken down to the relevant time horizon.

Maturity Period refers to the number of years after which the par value becomes payable to the bond-holder. Generally, corporate bonds have a maturity period of 7-10 years and government bonds 20-25 years.

Redemption Value is the amount the bond-holder gets on maturity. A bond may be redeemed at par, at a premium (bond-holder gets more than the par value of the bond) or at a discount (bond-holder gets less than the par value of the bond).

Market Value is the price at which the bond is traded in the stock exchange. Market price is the price at which the bonds can be bought and sold and this price may be different from par value and redemption value.

Types of Bonds: Bonds are of three types: (a) Irredeemable Bonds (also called perpetual bonds) (b) Redeemable Bonds (i.e., Bonds with finite maturity period) and (c) Zero Coupon Bonds.

Irredeemable Bonds or Perpetual Bonds: Bonds which will never mature are known as irredeemable or perpetual bonds. Indian Companies Acts restricts the issue of such bonds and therefore these are very rarely issued by corporate these days. In case of these bonds the terminal value or maturity value does not exist because they are not redeemable. The face value is known; the interest received on such bonds is constant and received at regular intervals and hence the interest receipts resemble a perpetuity. The present value (the intrinsic value) is calculated as:

$$V_0 = I/id$$

If a company offers to pay ` 70 as interest on a bond of ` 1,000 par value, and the current yield is 8%, the value of the bond is **70/0.08** which is equal to **` 875**

Redeemable Bonds: There are two types viz., bonds with annual interest payments and bonds with semi-annual interest payments.

Bonds with Annual Interest Payments

Basic Bond Valuation Model: The holder of a bond receives a fixed annual interest for a specified number of years and a fixed principal repayment at the time of maturity. The intrinsic value or the present value of bond can be expressed as:

$$V_0 \text{ or } P_0 = \sum^{n}_{t=1} I/(I + k_d)^n + F/(I + k_d)^n$$

Which can also be stated as follows:

$$V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)$$

Where V_0 = Intrinsic value of the bond

P_0 = Present Value of the bond

I = Annual Interest payable on the bond

F = Principal amount (par value) repayable at the maturity time

N = Maturity period of the bond

K_d = Required rate of return

Illustration 29. A bond whose face value is ` 100 has a coupon rate of 12% and a maturity of 5 years. The required rate of interest is 10%. What is the value of the bond?

Solution: Interest payable = 100 × 12% = ` 12

Principal repayment is ` 100

Required rate of return is 10%

$$\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$$

Value of the bond = 12 × PVIFA(10%, 5y) + 100 × PVIF(10%, 5y)

= 12 × 3.791 + 100 × 0.621

= 45.49 + 62.1

= ` 107.59

Illustration 30. Mr. Anant purchases a bond whose face value is ` 1,000, maturity period 5 years coupled with a nominal interest rate of 8%. The required rate of return is 10%. What is the price he should be willing to pay now to purchase the bond?

Solution: Interest payable = 1,000 × 8% = ` 80

Principal repayment is ` 1,000

Required rate of return is 10%

V_0 = I × PVIFA(kd, n) + F × PVIF(kd, n)

Value of the bond = 80 × PVIFA(10%, 5y) + 1,000 × PVIF(10%, 5y)

= 80 × 3.791 + 1,000 × 0.621

= 303.28 + 621

= ` 924.28

This implies that the company is offering the bond at ` 1,000 but is worth ` 924.28 at the required rate of return of 10%. The investor may not be willing to pay more than ` 924.28 for the bond today.

Bond Values with Semi-annual Interest Payment: In reality, it is quite common to pay interest on bonds semi-annually. With the effect of compounding, the value of bonds with semi-annual interest is much more than the ones with annual interest payments. Hence, the bond valuation equation can be modified as:

$$V_0 \text{ or } P_0 = \sum_{t=1}^{n} I/2(I + i_0/2)^n + F/(I + i_0/2)^{2n}$$

Where, V_0 = Intrinsic value of the bond

P_0 = Present value of the bond

I/2 = Semi-annual interest payable on the bond

F = Principle amount (par value) repayable at the maturity time

2n = Maturity period of the bond expressed in half-yearly periods

$k_0/2$ = Required rate of return semi-annually.

Example: A bond of ` 1,000 value carries a coupon rate of 10%, maturity period of 6 years. Interest is payable semi-annually. If the required rate of return is 12%, calculate the value of the bond.

Solution:

V_0 or P_0 = $\sum^{n}_{t=1}$ (I/2)(I + k_0/2)^n + F/(I + k_0/2)^{2n}

= (100/2)/(1 + 0.12/2)4 + 1,000/(1 + 0.12/2)4

= 50 × PVIFA(6%, 12y) + 1,000 × PVIF(6%, 12y)

= 50 × 8.384 + 1,000 × 0.497

= 419.2 +497

= ` 916.20

It is to be kept in mind that the required rate of return is halved (12%/2) and the period doubled (6y × 2) as the interest is paid semi-annually.

Valuation of Zero Coupon Bonds: In India Zero coupon bonds are alternatively known as Deep Discount Bonds. For close to a decade, these bonds became very popular in India because of issuance of such bonds at regular intervals by IDBI and ICICI. Zero-coupon bonds have no coupon rate, i.e. there is no interest to be paid out. Instead, these bonds are issued at a discount to their face value, and the face value is the amount payable to the holder of the instrument on maturity. The difference between the discounted issue price and face value is effective interest earned by the investor. They are called deep discount bonds because these bonds are long term bonds whose maturity some time extends up to 25 to 30 years.

Illustration 31. River Valley Authority issued Deep Discount Bond of the face value of ` 1,00,000 payable 25 years later, at an issue price of ` 14,600. What is the effective interest rate earned by an investor from this bond?

Solution: The bond in question is a zero coupon or deep discount bond. It does not carry any coupon rate. Therefore, the implied interest rate could be computed as follows:

Step 1. Principal invested today is ` 14,600 at a rate of interest of "r"% over 25 years to amount to ` 1,00,000.

Step 2. It can be stated as A = P_0 (1 + r)^n

1,00,000 = 14,600 (1 + r)^{25}

Solving for 'r', we get 1,00,000/14600 = (1 + r)^{25}

6.849 = (1 + r)^{25}

Reading the compound value (FVIF) table, horizontally along the 25 year line, we find 'r' equals 8%. Therefore, bond gives an effective return of 8% per annum.

Bond-yield Measures

Current Yield: Current yield measures the rate of return earned on a bond if it is purchased at its current market price and the coupon interest received.

Current Yield = Coupon Interest/Current Market Price

Illustration 32. Continuing with the same example above calculate the CY if the current market price is ` 920

Solution: CY = Coupon Interest/Current Market Price

= 80/920

= 8.7%

Yield to Maturity (YTM): It is the rate earned by an investor who purchases a bond and holds it till its maturity. The YTM is the discount rate equaling the present values of cash flows to the current market price.

Illustration 33. A bond has a face value of ` 1,000 with a 5 year maturity period. Its current market price is ` 883.4. It carries an interest rate of 6%. What shall be the rate of return on this bond if it is held till its maturity?

Solution:

V_0 or P_0 $= \sum^{n}_{t=1} (I/2)(I + k_d)^n + F/(I + k_0 d)^n$

OR

V_0 = I × PVIFA (kd, n) + F × PVIF(kd, n)

= 60 × PVIFA(Kd, 10) + 1,000 × PVIF(Kd,10) = 883.4

We obtain **10%** for kd

Illustration 34. A bond has a face value of ` 1,000 with a 9 year maturity period. Its current market price is ` 850. It carries an interest rate of 8%. What shall be the rate of return on this bond if it is held till its maturity?

Solution:

V_0 or P_0 $= \sum^{n}_{t=1} (I/2)(I + k_d)^n + F/(I + k_0 d)^n$

OR

V_0 = I × PVIFA (kd, n) + F × PVIF(kd, n)

= 80 × PVIFA(Kd%, 9) + 1,000 × PVIF(Kd%, 9) = 850

To find out the value of Kd, trial an error method is to be followed. Let us therefore start the value of Kd to be 12% and the equation now looks like = 80 × PVIFA(12%, 9) + 1,000 × PVIF(12%, 9) = 850.

Let us now see if LHS equals RHS at this rate of 12%. Looking at the tables we get LHS as 80 × 5.328 + 1,000 × 0.361 = ` 787.24.

Since this value is less than the value required on the RHS, we take a lesser discount rate of 10%. At 10%, the equation is = 80 × PVIFA(10%, 9) + 1,000 × PVIF(10%, 9) = 850.

Let us now see if LHS equals RHS at this rate of 11%. Looking at the tables we get LHS as 80 × 5.759 + 1,000 × 0.424 = ` 884.72.

We now understand that Kd clearly lies between 10% and 12%. We shall interpolate to find out the true value of Kd.

10% + {(884.72 – 850)/(884.72 – 787.24)} × (12% – 10%)

10% + (34.72/97.48) × 2

10% + 0.71

Therefore **Kd = 10.71%**

An approximation: The following formula may be used to get a rough idea about **Kd** as Trial and Error Method is a very tedious procedure and requires lots of time. This formula can be used as a ready reference formula.

YTM = {I + (F – P)/n}/{(F + P)/2}

Where YTM = Yield to Maturity

I = Annual interest payment

F = Face value of the bond

P = Current market price of the bond

N = Number of years to maturity.

Illustration 35. A company issues a bond with a face value of 5,000. It is currently trading at ` 4,500. The interest rate offered by the company is 12% and the bond has a maturity period of 8 years. What is YTM?

Solution: YTM = {I + (F – P)/n}/{(F + P)/2}

= 600 + {(5000 – 4500)/8}/{(5000 + 4500)/2}

= {600 + 62.5}/4750

= **13.94%**

Bond Value Theorems

The following factors affect the bond values:

- Relationship between the required rate of interest (Kd) and the discount rate.
- Number of years to maturity.
- YTM

Relationship between the required rate of interest (Kd) and the discount rate:

- When Kd is equal to the coupon rate, the intrinsic value of the bond is equal to its face value, that is, if Kd = coupon rate, then value of bond = face value.
- When Kd is greater than the coupon rate, the intrinsic value of the bond is less than its face value, that is, if Kd > coupon rate, then value of bond < face value.
- When Kd is lesser than the coupon rate, the intrinsic value of the bond is greater than its face value, that is, if Kd < coupon rate, then value of bond > face value.

Illustration 36. Sugam industries wishes to issue bonds with ` 100 as par value, coupon rate 12% an YTM 5 years. What is the value of the bond if the required rate of return of an investor is 12%, 14% and 10%.

If Kd is 12%, $\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 12 \times PVIFA(12\%, 5) + 100 \times PVIF(12\%, 5)$

$= 12 \times 3.605 + 100 \times 0.567$

$= 43.26 + 56.7$

= ` 99.96 or ` 100

If Kd is 14%, $\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 12 \times PVIFA(14\%, 5) + 100 \times PVIF(14\%, 5)$

$= 12 \times 3.433 + 100 \times 0.519$

$= 41.20 + 51.9$

= **` 93.1**

If Kd is 10%, $\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 12 \times PVIFA(10\%, 5) + 100 \times PVIF(10\%, 5)$

$= 12 \times 3.791 + 100 \times 0.621$

$= 45.49 + 62.1$

= **` 107.59**

Number of Years to Maturity

- When Kd is greater than the coupon rate, the discount on the bond declines as maturity approaches.
- When Kd is less than the coupon rate, the premium on the bond declines as maturity approaches.

To show the effect of the above, consider a case of a bond whose face value is ` 100 with a coupon rate of 11% and a maturity of 7 years.

If Kd is 13%, then, $\mathbf{V_0 = I - PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 11 \times PVIFA(13\%, 7) + 100 \times PVIF(13\%, 7)$

$= 11 \times 4.423 + 100 \times 0.425$

$= 48.65 + 42.50$

= ` 91.15

After 1 year, the maturity period is 6 years, the value of the bond is

$\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 11 \times PVIFA(13\%, 6) + 100 \times PVIF(13\%, 6)$

$= 11 \times 3.998 + 100 \times 0.480$

$= 43.98 + 48$

= ` 91.98.

We see that the discount on the bond gradually decreases and value of the bond increases with the passage of time at **Kd** being a higher rate than the coupon rate.

Continuing with the same example above, let us see the effect on the bond value if required rate is 8%.

If Kd is 8%, $\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 11 \times PVIFA(8\%, 7) + 100 \times PVIF(8\%, 7)$

$= 11 \times 5.206 + 100 \times 0.583$

$= 57.27 + 58.3$

= ` **115.57**

One year later, Kd at 8%,

$\mathbf{V_0 = I \times PVIFA(kd, n) + F \times PVIF(kd, n)}$

$= 11 \times PVIFA(8\%, 6) + 100 \times PVIF(8\%, 6)$

$= 11 \times 4.623 + 100 \times 0.630$

$= 50.85 + 63$

= ` **113.85**

For a required rate of return of 8%, the bond value decreases with passage of time and premium on bond declines as maturity approaches.

YTM: YTM determining the market value of the bond, the bond price will fluctuate to the changes in market interest rates. A bond's price moves inversely proportional to its YTM.

Valuation of Shares: A company's shares may be categorized as (a) Ordinary or Equity shares and (b) Preference shares. The returns these shareholders get are called dividends. Preference shareholders get a preferential treatment as to the payment of dividend and repayment of capital in the event of winding up. Such holders are eligible for a fixed rate of dividends. Some important features of preference and equity shares.

- **Dividends:** Rate is fixed for preference shareholders. They can be given cumulative rights, that is, the dividend can be paid off after accumulation. The dividend rate is not fixed for equity shareholders. They change with an increase or decrease in profits. During years of big profits, the management may declare a high dividend. The dividends are not cumulative for equity shareholders, that is, they cannot be accumulated and distributed in later years. Dividends are not taxable.
- **Claims:** In the event of the business closing down, the preference shareholders have a prior claim on the assets of the company. Their claims shall be settled first

and the balance if any will be paid off to equity shareholders. Equity shareholders are residual claimants to the company' income and assets.

- **Redemption:** Preference shares have a maturity date on which day the company pays off the face value of the share to the holders. Preference shares can be of two types – redeemable and irredeemable. Irredeemable preference shares are perpetual. Equity shareholders have no maturity date.
- **Conversion:** A company can issue convertible preference shares and not vice versa. After a particular period as mentioned in the share certificate, the preference shares can be converted into ordinary shares.

Valuation of Preference Shares: Preference shares, like bonds carry a fixed rate of dividend/return. Symbolically, this can be expressed as:

$$P_0 = Dp/\{1 + Kp)^n\} + P_n/\{(1 + Kp)^n\} \text{ OR}$$

$$P_0 = Dp \times PVIFA(Kp, n) + P_n \times PVIF(Kp, n)$$

Where P_0 = Price of the share

Dp = Dividend on preference share

Kp = Required rate of return on preference share

N = Number of years to maturity

Valuation of Ordinary Shares: People hold common stocks for two reasons – to obtain dividends in a timely manner and to get a higher amount when sold. Generally, shares are not held in perpetuity. An investor buys the shares, holds them for some time during which he gets dividends and finally sells it off to get capital gains. The value of a share which an investor is willing to pay is linked with the cash inflows expected and risks associated with these inflows. Intrinsic value of a share is associated with the earnings (past) and profitability (future) of the company, dividends paid and expected and future definite prospects of the company. It is the economic value of a company considering its characteristics, nature of business and investment environment.

Dividend Capitalization Model: When a shareholder buys a share, he is actually buying the stream of future dividends. Therefore the value of an ordinary share is determined by capitalizing the future dividend stream at an appropriate rate of interest. So under the dividend capitalization approach, the value of an equity share is the discounted present value of dividends received plus the present value of the resale price expected when the share is disposed. Two assumptions are made to apply this approach:

- Dividends are paid annually.
- First payment of dividend is made after one year the equity share is bought.

Single Period Valuation Model: This model holds well when an investor holds an equity share for one year. The price of such a share will be:

$$P_0 = \frac{D_1}{(1+Ke)} + \frac{P_1}{(1+Ke)}$$

Where P_0 = Current market price of the share

D_1 = Expected dividend after one year

P_1 = Expected price of the share after one year

Ke = Required rate of return on the equity share

Illustration 37. Gammon India Ltd.'s share is expected to touch ` 450 one year from now. The company is expected to declare a dividend of ` 25 per share. What is the price at which an investor would be willing to buy if his required rate of return is 15%?

Solution: P_0 = D_1/(1 + Ke) + P_1/(1 + Ke)

= {25/(1 + 0.15)} + {450/(1 + 0.15)}

= 21.74 + 391.30

= ` 413.04 is the price he is willing to pay today

Multi-period Valuation Model: An equity share can be held or an indefinite period as it has no maturity date, in which case the value of a price at time zero is:

$$P_0 = D_1/(1 + Ke)^1 + D_2/(1 + Ke)^2 + D_3/(1 + Ke)^3 + \ldots\ldots.. + D_\infty/(1 + Ke)^\infty$$

OR

$$P_0 = \sum\nolimits^{n}_{t=1} \frac{D_1}{\{(1+Ke)^\infty\}}$$

Where P_0 = Current market price of the share

D_1 = Expected dividend after one year

P_1 = Expected price of the share after one year

D_∞ = Expected dividend at infinite duration

Ke = Required rate of return on the equity share.

The above equation can also be modified to find the value of an equity share for a finite period.

$$P_0 = D_1/(1 + Ke)^1 + D_2/(1 + Ke)^2 + D_3/(1 + Ke)^3 + \ldots\ldots.. + D_\infty/(1 + Ke)^\infty + P_\infty/(1 + Ke)^\infty$$

$$P_0 = \sum\nolimits^{n}_{t=1} D_\infty/\{(1 + Ke)^\infty\} + P_\infty/\{(1 + Ke)^\infty\}$$

We can come across three instances of dividends in companies:

- Constant dividends
- Constant growth of dividends
- Changing growth rates of dividends.

Valuation with constant dividends: If constant dividends are paid year

After year, then P0 = $D_1/(1 + Ke)^1 + D_2/(1 + Ke)^2 + D_3/(1 + Ke)^3 + \ldots\ldots + D_\infty/(1 + Ke)^\infty$

Simplifying this we get P = D/Ke

Valuation with constant growth in dividends: Here we assume that dividends tend to increase with time as and when businesses grow over time. If the increase in dividend is at a constant compound rate, then **$P_0 = D_1$/Ke-g**, where g stands for growth rate.

Illustration 38. Sagar automobiles Ltd.'s share is traded at ` 180. The company is expected to grow at 8% per annum and the dividend expected to be paid off is ` 8. If the rate of return is expected to be 12%, what is the price of the share one would be expected to pay today?

Solution: P_0 = D_1/Ke-g

= 8/0.12 – 0.08

= **` 200.**

Illustration 39. Monica labs is expected to pay ` 4 as dividend per share next year. The dividends are expected to grow perpetually@8%. Calculate the share price today if the market capitalization is 12%.

Solution: P_0 = D_1/Ke-g

P_0 = 4/(0.12 – 0.08)

= ` 100

Valuation with Variable Growth in Dividends: Some firms may not have a constant growth rate of dividends indefinitely. There are periods during which the dividends may grow super-normally, that is, the growth rate is very high when the demand for the company's products is very high. After a certain period of time, the growth rate may fall to normal levels when the returns fall due to fall in demand for products (with competition setting in or due to availability of substitutes). The price of the equity share of such a firm is determined in the following manner:

Step 1. Expected dividend flows during periods of supernormal growth is to be considered and present value of this is to be computed with the following equation:

$$P_0 = \sum^{n}_{t=1} D_n/(1 + Ke)^n$$

Value of the share at the end of the initial growth period is calculated as:

$P_n = (D_{n+1})/(Ke - gn)$ (constant growth model). This is discounted to the present value and we get:

$(D_{n+1})/(Ke - gn) \times 1/(1 + Ke)^n$

Add both the present value composites to find the value P0 of the share, that is, **$P_0 = \sum^{\infty}_{t=1} D_n/(1 + Ke)^n + (D_{n+1})/(Ke - gn) \times 1/(1 + Ke)^n$**

Illustration 40. Souparnika Pharma's current dividend is ` 5. It expects to have a supernormal growth period running to 5 years during which the growth rate would be

25%. The company expects normal growth rate of 8% after the period of supernormal growth period. The investors' required rate of return is 15%. Calculate what the value of one share of this company is worth.

Solution: $D_0 = 5$, n = 5y, ga (supernormal growth) = 25%, gn (normal growth) = 8%, Ke = 14%

Step I: $P_0 = \sum_{t=1}^{\infty} D_n/(1 + Ke)^n$

$D1 = 5\ (1.25)^1$

$D2 = 5\ (1.25)^2$

$D3 = 5\ (1.25)^3$

$D4 = 5\ (1.25)^4$

$D5 = 5\ (1.25)^5$

The present value of this flow of dividends will be:

$5(1.25)/(1.15) + 5(1.25)^2/(1.15)^2 + 5(1.25)^3/(1.15)^3 + 5(1.25)^4/(1.15)^4 + 5(1.25)^5/(1.15)^5$

$5.43 + 5.92 + 6.42 + 6.98 + 7.63 = 32.38$

Step II: $P_n = (D_{n+1})/(Ke - g)$

$P5 = D6/Ke - gn$

$= D5(1 + gn)/Ke - gn$

$= \{5(1.25)^5\ (1 + 0.08)\}/(0.15 - 0.08)$

$= 15.26(1.08)/0.07$

$= 16.48/0.07$

$= 235.42$

The discounted value of this price is $235.42/(1.15)^5$ = ` 117.12

Step III: $P_0 = \sum_{t=1}^{\infty} D_n/(1 + Ke)^n + (D_{n+1})/(Ke - gn) \times 1/(1 + Ke)^n$

The value of the share is ` 32.38 + ` 117.12 = ` 149.50

Other Approaches to Equity Valuation

In addition to the dividend valuation approaches discussed in the previous section, there are other approaches to valuation of shares based on **"Ratio Approach".**

Book Value Approach: The book value per share (BVPS) is the *net worth of the company divided by the number of outstanding equity shares.* Net worth is represented by the sum total of paid up equity shares, reserves and surplus. Alternatively, this can also be calculated as the *amount per share on the sale of the assets of the company at their exact book value minus all liabilities including preference shares.*

Illustration 41. A One Ltd. has total assets worth ` 500 Cr., liabilities worth ` 300 Cr., and preference shares worth ` 50 Cr. and equity shares numbering 10 lakhs

Solution: The BVPS is ` 150 Cr/10 lakhs = ` 150

BVPS does not give a true investment picture. This relies on historical book values than the company's earning potential.

Liquidation Value: The liquidation value per share is calculated as:

{(Value realized by liquidating all assets) – (Amount to be paid to all Crs and Pre SH)} divided by Number of outstanding shares.

In the above example, if the assets can be liquidated at ` 450 Cr., the liquidation value per share is (450Cr – 350Cr)/10 lakh shares which is equal to ` 1,000 per share.

Price Earnings Ratio: The price-earnings ratio reflects the amount investors are willing to pay for each rupee of earnings.

Expected Earnings per Share = (Expected PAT) – (Preference dividend)/Number of Outstanding Shares. Expected PAT is dependent on a number of factors like sales, gross profit margin, depreciation and interest and tax rate. The P/E ratio is also to consider factors like growth rate, stability of earnings, company size, company management team and dividend pay-out ratio.

P/E ratio = (1 – b)/r – (ROE × b)

Where 1-b is dividend pay out ratio

r is required rate of return

ROE × b is expected growth rate.

Illustration 42. The current price of a Ashok Leyland share is ` 30. The company is expected to pay a dividend of ` 2.50 per share which goes up annually at 6%. If an investor's required rate of return is 11%, should he buy this share or not? Advise.

Solution: P = D_1(1 + g)/Ke – g = 2.5(1 + 0.06)/0.11 – 0.06 = **` 53.** The investor should certainly buy this share at the current price of ` 30 as the valuation model says the share is worth ` 53.

Illustration 43. A bond with a face value of ` 100 provides an annual return of 8% and pays ` 125 at the time of maturity, which is 10 years from now. If the investor's required rate of return is 12%, what should be the price of the bond?

Solution: P = Int. × PVIFA(12%, 10y) + Redemption price × PVIF(12%, 10y)

= 8 × PVIFA(12%, 10y) + 125 × PVIF(12%, 10y)

= 8 × 5.65 + 125 × 0.322

= 45.2 + 40.25

= **` 85.45**

The price of the bond should be ` 85.45.

Illustration 44. The bond of Silicon Enterprises with a par value of ` 500 is currently traded at ` 435. The coupon rate is 12% with a maturity period of 7 years. What will be the yield to maturity?

Solution: r = I + {(F – P)/n}/(F + P)/2

= 60 + {(500 – 435)/7}/(500 + 435)/2

= **15.03%**

Illustration 45. The share of Megha Ltd is sold at ` 500 a share. The dividend likely to be declared by the company is ` 25 per share after one year and the price one year hence is expected to be ` 550. What is the return at the end of the year on the basis of likely dividend and price per share?

Solution: Holding period return = (D1 + Price gain/loss)/purchase price

= (25 + 50)/500 = **15%**

Illustration 46. A bond of face value of ` 1000 and a maturity of 3 years pays 15% interest annually. What is the market price of the bond if YTM is also 15%?

Solution: P = Int. × PVIFA(15%, 3y) + Redemption value × PVIF(15%, 3y)

P = 150 × 2.283 + 1000 × 0.658

P = 342.45 + 658 = **` 1,000.45**

Illustration 47. A perpetual share pays an annual dividend of ` 15 on a face value of ` 100 and the rate of return required by investors on such investments is 20%. What should be the market price of the preference share?

Solution: Expected yield = Expected income/current market price

Expected yield = 15/0.2 = **` 75**

EXERCISE

Self Assessment Questions 1

1. The important factors contributing to time value of money are ____________, ____________ and ____________.
2. During periods of inflation, a rupee has a ____________ than a rupee in future.
3. As future is characterized by uncertainty, individuals prefer ____________ consumption to ____________ consumption.
4. There are two methods by which time value of money can be calculated by ____________ and ____________ techniques.

Self Assessment Questions 2

1. _____________ is created out of fixed payments each period to accumulate to a future sum after a specified period.
2. The _____________ of a future cash flow is the amount of the current cash that is equivalent to the investor.
3. An annuity for an infinite time period is called _____________.
4. The reciprocal of the present value annuity factor is called _____________.

Self Assessment Questions 3

1. _____________ is the minimum value the company accepts if it sold its business.
2. _____________ per share is generally higher than the book value per share for profitable and growing firms.
3. Bonds issued by _____________ are secured and those issued by private sector companies may be _____________ or _____________.
4. _____________ is the rate earned by an investor who purchases a bond and holds it till its maturity.
5. When Kd is lesser than the coupon rate, the value of the bond is _____________ than its face value.
6. _____________ of a share is associated with the earnings (past) and profitability (future) of the company, dividends paid and expected and future definite prospects of the company.
7. The _____________ is the net worth of the company divided by the number of outstanding equity shares.

Answers to SAQs

Self Assessment Questions 1

1. Investment opportunities, preference for consumption, risk.
2. Higher purchasing power
3. Current and future
4. Compounding and discounting

Self Assessment Questions 2

1. Sinking fund
2. Present Value PV
3. Perpetuity
4. Capital Recovery Factor.

Self Assessment Questions 3

1. Liquidation value
2. Market value
3. Government agencies, secured or unsecured
4. Yield to Maturity
5. Greater
6. Intrinsic value
7. Book value per share (BVPS)

Self Assessment Questions 3

(a) When compounding is done more than annually, the effective rate of interest is ____________.

(i) Greater than the nominal rate of interest
(ii) Lower than nominal rate of interest
(iii) Equal to nominal rate of interest

(**Ans:** i)

(b) Which provides money with its time value?

(i) Investment
(ii) Interest rate
(iii) Market rates
(iv) Currency rates

(**Ans:** ii)

(c) When payments are made at the end of each year, it is known as ____________ annuity.

(i) Annuity due
(ii) Ordinary annuity
(iii) Perpetuity
(iv) Fixed annuity

(**Ans:** ii)

Terminal Questions

1. If you deposit ` 10,000 today in a bank that offers 8% interest, in how many years will this amount double?
2. An employee of a bank deposits ` 30,000 into his PF A/c at the end of each year for 20 years. What is the amount he will accumulate in his PF at the end of 20 years, if the rate of interest given by PF authorities is 9%?

3. A person can save ____________ annually to accumulate ` 4,00,000 by the end of 10 years, if the saving earns 12%.
4. Mr. Vinod has to receive ` 20,000 per year for 5 years. Calculate the present value of the annuity assuming he can earn interest on his investment at 10% p.a.
5. Aparna invests ` 5,000 at the end of each year at 10% interest p.a. What is the amount she will receive after 4 years?

Answers to Terminal Questions

1. (Hint: Use rule of 72 and 69)
2. 30,000 × FVIFA(9%, 20Y) = 30,000 × 51.160 = **` 15,34,800**
3. A × FVIFA(12%, 10y) = 4,00,000 which is 4,00,000/17.549 = **` 22,795**
4. 20,000 × PVIFA(105, 5y) = 20,000 × 3.791 = **` 75,820**
5. 5,000 × FVIFA(10%, 4y) = 5,000 × 6.105 = **` 23,205**

Terminal Questions

1. What should be price of a bond which has a par value of ` 1,000 carrying a coupon rate of 8% and having a maturity period of 9 years? The required rate of return of the investor is 12%.
2. A bond of ` 1,000 value carries a coupon rate of 10% and has a maturity period of 6 years. Interest is payable semi-annually. If the required rate of return is 12%, calculate the value of the bond.
3. A bond whose par value is ` 500 bearing a coupon rate of 10% and has a maturity of 3 years. The required rate of return is 8%. What should be the price of the bond?
4. If the current year's dividend is ` 24, growth rate of a company is 10% and the required return on the stock is 16%, what is the intrinsic value of the stock?
5. If a stock is purchased for ` 120 and held for one year during which time ` 15 dividend per share is paid and the price decreases to ` 115, what is the nominal return on the share?

Answers to Terminal Questions

1. P = Int. × PVIFA(12%, 9y) + Redemption price × PVIF(12%, 10y)

   80 × PVIFA(12%, 9) + 1,000 × PVIF(12%, 9y)

   80 × 5.328 + 1,000 × 0.361

   426.24 + 361 **= ` 787.24**
2. 50 × PVIFA(6% + 12y) + 1,000 × PVIF(6% + 12y)

   50 × 8.384 + 1,000 × 0.497 **= ` 916.2**
3. P = Int. × PVIFA(8%, 3y) + Redemption price × PVIF(8%, 3y)

   50 × 2.577 + 500 × 0.794

   128.85 + 397 = **` 525.85**

4. Intrinsic value = 24 {(1 + 0.1)}/0.16 – 0.1 = **` 440**
5. Holding period return = (D1 + Price gain/loss)/purchase price

   {15 + (-5)}/120 = **8.33%**

Terminal Questions

1. Ramesh deposited ` 4,000 for 3 years period at 12% interest which is credited at the end of every six months. What will be the total amount credited to Ramesh's account at the end of 3 years?
2. Ganesh plans to send his son for higher studies in America after 5 years. He expects the cost of the study to be ` 4,00,000. How much should he save annually to have a sum of ` 4,00,00 at the end of 5 years, if the interest rate is 9%?
3. ICICI Bank promises to give you ` 5,000 after 10 years in exchange of ` 2,000 today. What is the interest rate involved in this offer?
4. Arvind wants to invest @ 8% p.a. compound interest, a such amount as will amount to ` 50,000 at the end of three years. How much should he invest?
5. A company has advertised for deposits from the public. If you deposit ` 1,000 now, you receive ` 1,464 at the end of 4 years or ` 1,611 at the end of 5 years. What rates of interest is the company paying?
6. Four equal annual payments of ` 4,000 are made into a deposit account that pays 8 per cent per year. What would be the future value of this annuity at the end of 6 years?
7. You can save ` 20,000 a year for 5 years and ` 3,000 a year for 10 years thereafter. What will these savings cumulate to at the end of 15 years if the rate of interest is 10 per cent?
8. Find out the present value of a debenture from the following:

   Face value of Debenture ` 1,000

   Annual Interest Rate 15%

   Expected return 12%

   Maturity Period 5 years

   (Present value of Re. 1 at 12% are, 0.8929, 0.7972, 0.7118, 0.6355, 0.5674)

   [**Ans:** PV_d = I (PVAF) + F (DF) = 1,108.12]
9. The share of Ridhi Ltd. (` 10) was quoting at ` 102 on 1.04.2002 and the price rose to ` 132 on 1.04.2005. Dividends were received at 10% on 30th June each year. Cost of Funds was 10% is it worthwhile investment, considering the time value of money. [**Ans:** NPV -0.382 is negative, Hence, it is not a wise investment.]
10. The future value of an amount invested or borrowed at a given rate of interest can be calculated if the maturity period is given. Suppose a deposit of ` 10,000 gets 10 per cent interest compounded annually for a period of 3 years, the future value will be? [**Ans:** ` 13,310]

11. Satish deposits ` 1,00,000 with a bank which pays 8 per cent interest compounded annually, for a period of 2 years how much he will get at maturity ?
[**Ans:** ` 1,16,640]

12. CSK deposits ` 10,000 with a bank at 12% interest compounded quarterly .How much amount he will get after a period of 6 years? [**Ans:** Rs .20,328]

13. Four equal annual payments of ` 5,000 are made into a deposit account that pays 8 per cent interest per year. What is the future value of this annuity at the end of 4 years. [**Ans:** ` 22,530.50]

14. A is due to receive ` 10,000 at the end of 5 years. Since A is in need of Money Immediately, He wants to sell his Interest to B. B wants a return of 10% per annum on his investment. How much should he pay A? [**Ans:** ` 6,209]

15. Krishnamurthy has inherited ` 1,000 a year for the next 20 years. First payment being made in one year's time. However, he is in need of money immediately and would like to sell his income to a buyer who would pay him the right price. Assume that the current market rate of interest is 9%:
    (a) What should be the right price he should accept
    (b) How much of his income should he sell if he wants only ` 2,500 at present
    (c) If you were interested in buying the income but, if you had only ` 5,000 to invest what would you do?

[**Ans:** (a) ` 9128.50, (b) ` 726.13, (c) ` 452.26]

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